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POSITIVE/NEGATIVE FEEDBACK IN AMPLIFICATION AND CONTROL

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This article is based upon a talk given at the Montreal Meeting of the AAAS December, 1964, before the session on Positive Feedback of the Society of General Systems Research, and rewritten as a tribute to George Philbrick on his recent birthday.

Let us first take a very quick and narrow look at a number of applications of positive and negative feedback in conventional amplification and control art. Then let us reflect upon the generality and implications of these developments.

The first figure shows a general functional element in the presence of two possible feedbacks, one having a positive sign sense and one having a negative, the sign sense being clearly established relative to simple summation. Again, I believe you will understand with me that the complexity in more elaborate systems arises very often due to the absence of a well-defined summing point. Here, in all cases I am going to consider, we shall assume a very simple algebraic summing point. I shall return to this point at the close.

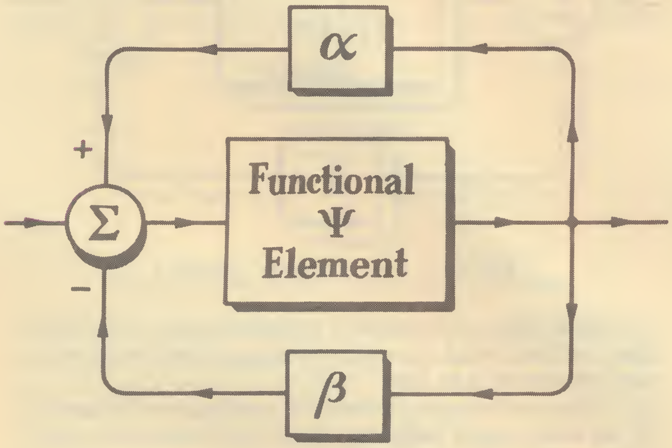


Figure 1.

I would now like to talk about what is inside the Ψ-box, and to a lesser extent what we are going to assume inside the boxes labelled "alpha" and "beta." Alpha I will consistently use for the operator on the positive feedback, beta for the negative feedback.

Figure 2 is referred to by some of my students as "Paynter's window." It represents a simple classification of functional elements, or operators, with respect to two attributes: whether they are dynamic or static, and whether they are nonlinear or linear. Thus Ψ represents a nonlinear dynamic operator, or functional element; Φ a nonlinear static element; *F*, a linear dynamic element; *K*, a linear static element.

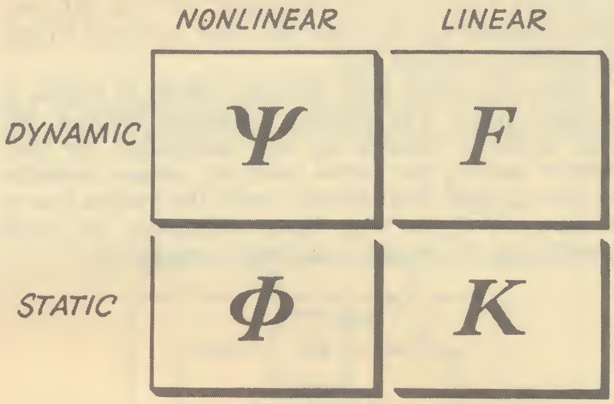


Figure 2. Classes of Functional Elements

Generally, the operators may be, if you please, operators with many inputs and many outputs. Our graphing does not necessarily imply a single input and a single output. An important observation is that we can get to the simplest element, namely a linear static operator, by two possible reductions: first, to linearity, then to *stasis* (or if you please, to the static condition); or alternatively, via a nonlinear path to a static condition, and then to the linear condition.

In Figure 3, you will see why I wished to discuss functional operators. I would like you for a moment to imagine each square being any one of the above operators (Ψ, Φ, *F*, *K*). We will here consider only the single input, single output case, just so we can be very specific. The little circles here stand for summation, so this "O" element is not an ambiguous or undefined operation, but rather a clearly defined *summation:* a special linear static operator. The boxes labelled alpha and beta for the moment could still represent two

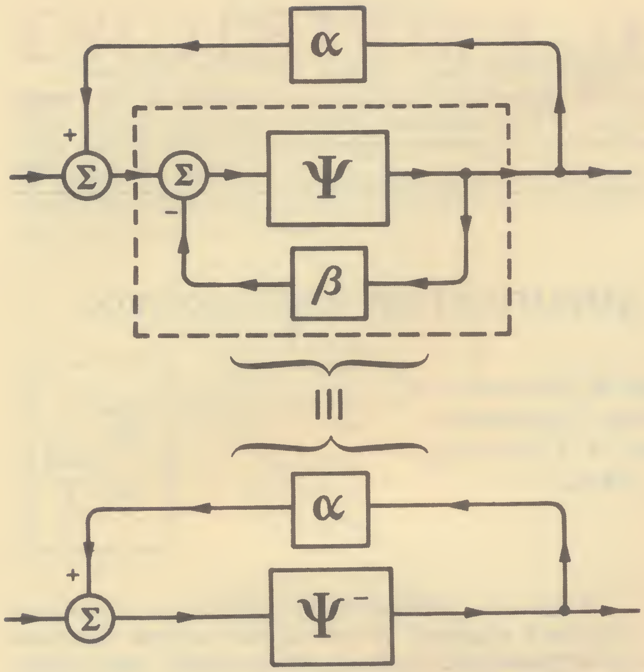


Figure 3.

other general operators, and the precise intent of the picture is to show that a simple reduction equivalence exists: namely, that by redefining the boundary of the functional operator one can define the operator *inside* the dotted box as a new operator, (Ψ^-) so labelled *minus* because the operator has been included within the *negative* feedback loop.

Now an exactly dual situation occurs as shown in the next figure. For obvious symmetry reasons one could similarly enclose the box embracing the plus operator, leaving the minus feedback element outside, and refer to that total element inside the broken line as psi plus ($\Psi+$), in turn being negatively fed back through beta, the negative feedback operator.

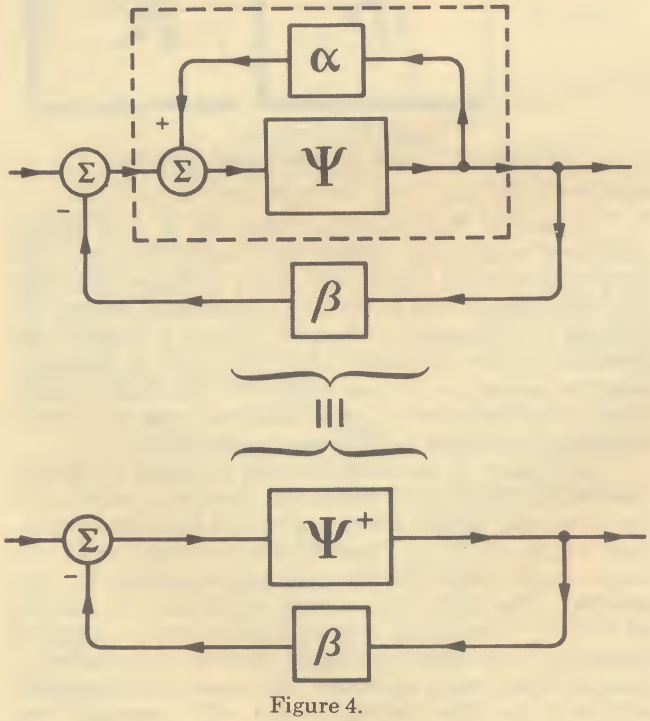


Figure 4.

There are two points of interest here, again relative to the vastly more complex systems of Nature. First, we are obviously exploiting the associative and dissociative properties of summation. Clearly, we can not perform this conceptual operation, unless we can *split* the single summation operation (dissociate it) and make it into two. Therefore, as depicted, we can perform this sequence of transformations in two alternative fashions.

Second, the purpose of such subtle considerations is to show that these systems can be classified into dual pairs. One may always associate pairs of systems, one with a positive feedback and the other with a negative feedback, and, further, this conjugation can be done in such a way that the resultant system is in effect an identity transformation. In other words, if you pass through both transformations, you would end up with the same system that you had at the start: alpha equals beta.

Thus is suggested the usefulness of the pairing of positive and negative feedback into dual schemes.

Static Operators

Figure 5 is the first of a series of several examples, each one evolving to slightly more sophisticated situations.

The first illustration exemplifies the genesis of degenerative or negative feedback, as a useful engineering concept. During the mid 1920's, and associated in particular with a patent originally issued to Black of the Bell System, was the idea to use negative feedback around as large a forward path gain as could be accomplished. If you take an infinite gain operator of any sort, as the basic functional operator, feed it back with a coefficient of *minus* one, (i.e., unity negative feedback), the output is simply a replica of the input signal. And this is what is meant here by the designation unit follower element.

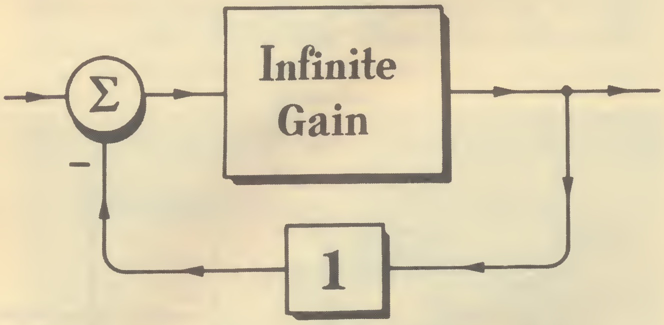


Figure 5. Unit Follower Element

Now this is a negative transformation. We might then be interested in seeing what positive transformation corresponds to this.

Here is its mate, or positive conjugate (Figure 6). If we take a unit follower, feed it back regeneratively with a plus sign, we will get an infinite gain element, and simple algebra may verify this. This represents a good deal of what common parlance refers to when speaking of positive feedback, referring ultimately to the fact that this is an explosive situation. Whatever input you may have, there will be infinitely more output, the regeneration being understood — indeed defined — as occurring instantaneously and immediately.

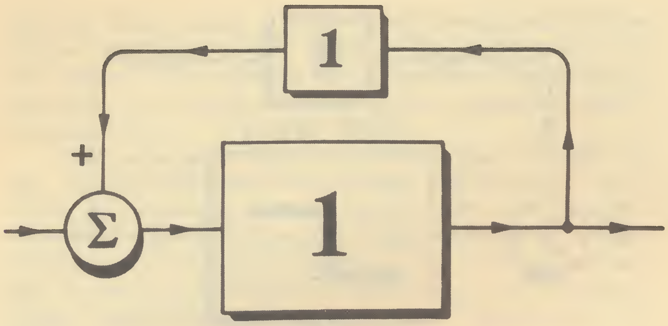


Figure 6. Infinite Gain Element

In Figure 7 is the first of another couple. This represents a slightly more complex marriage, and I will now take the positive before the negative (male, first, if you please). Here we have a simple proportional element, but one which saturates symmetrically (for simplicity). If this be fed back with unit positive gain, one will obtain the signum function. This yields plus one for positive input, minus one for negative input, zero for no input at all. And if now we look at the mate of this male we know immediately her attributes. (Figure 8).

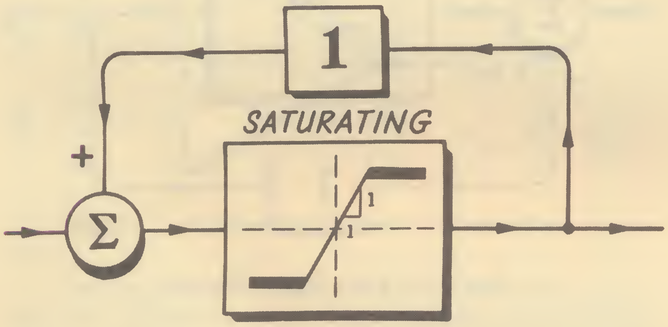


Figure 7. Signum Element

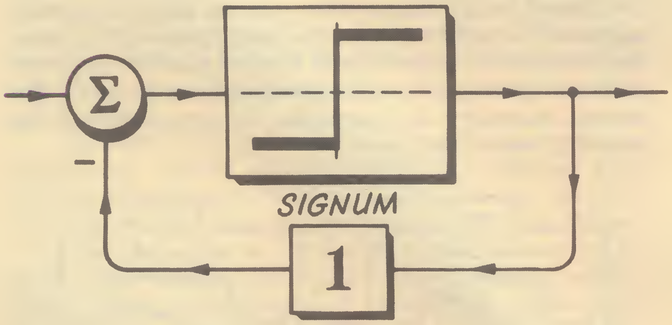


Figure 8. Saturating Proportional Element

The answer is that if we take the signum element and feed it back negatively with a gain of one, we must end up with what we started with, namely: saturating proportional response. We now see there is quite some value in considering these as pairs, because if we do not know what one side of the pair is, we can immediately determine the result if we know the spouse. We can infer from the original construction that this must always be true. No matter what reduction process we use to find the answer, we *must* obtain as a result our original saturating proportional element.

Now another advantage of this concept is shown in Figure 9, where we take this same element, the signum function, but now fed back by an additional gain of one. You will shortly see why I speak of it as an *additional* positive feedback. What we then get is a simple hysteresis element. It has a memory, or what I call *mnesis,* associated with it; and it is a static functional operator in the sense that whenever the input stops changing the output stops changing; but it is a mnesic static functional element because it does have memory associated with it. It can remember what the history was; it has a one-bit memory, but that one bit is very important. In fact, if it were not for that one bit none of the wall toggle switches would work, because they store the previous state in a simple toggling action, or hysteresis.

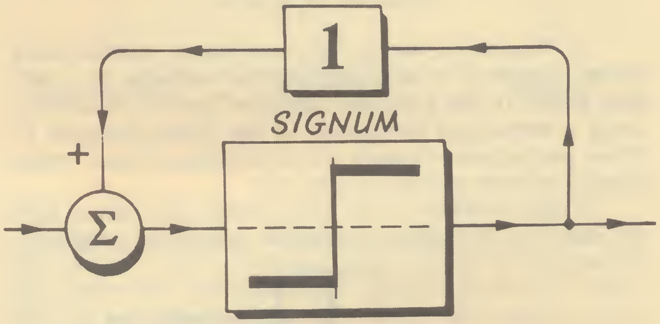


Figure 9. Hysteresis Element

The interrelation of these operations is best shown in Figure 10. In Figure 10a is the element we start with, a saturating proportional element. If now we feed it back with a gain of plus one (Figure 10b), we will get the signum function. If we feed it with a gain of plus two (Figure 10c), we will get a hysteretic element.

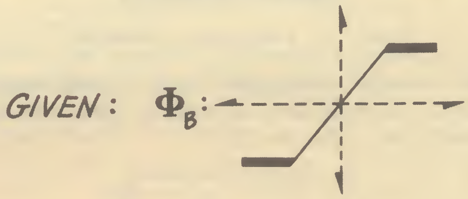


Figure 10 a.

THEN:

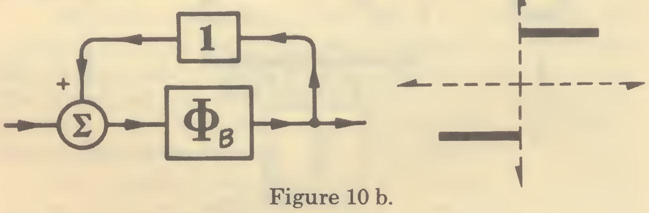


Figure 10 b.

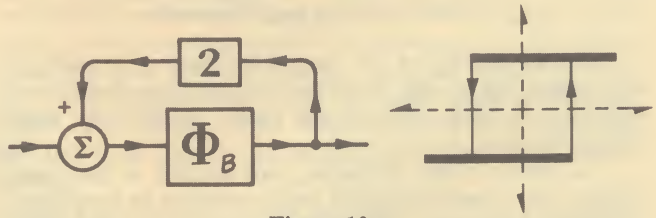


Figure 10 c.

This helps to demonstrate the fact that the form of the resultant functional operator depends upon the strength of the feedback — not simply the sign of the

feedback. Most behavioral scientists and most hard scientists — physical scientists- and engineers — would tend to agree that saturation is distinct from signum, and distinct from hysteresis. There is a tendency to believe that such violent changes in behavior must result from three fundamentally *different* operators. Yet we note they result from nothing more than strength zero positive feedback, strength one, and strength two, applied to the same underlying operator. With the more general situation in which the strength of the feedback gain is varied with the level of the output signal, one would observe a characteristic change in functional behavior with the level of the input signal.

Dynamic Operators

In Figure 11, we use the system of schematizing linear operators by representing the response to a unit step input. In the previous "window" we see this represents a movement *horizontal* to the linear operator *F*. This particular operator *F* is simply a first order passive process called the unit lag.

If this process is fed back positively with a gain of one, it will become a simple temporal integrator.

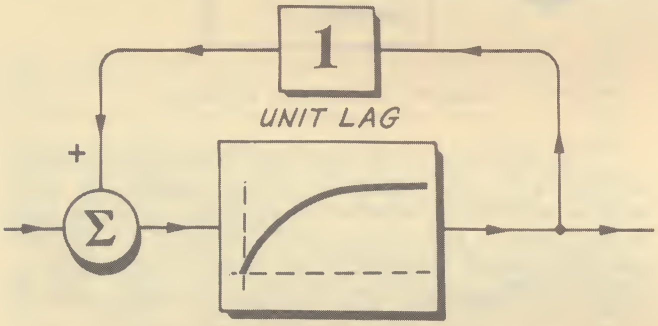


Figure 11. Integrating Element

Now take an integrator, feed it back negatively with a gain of one, and, of course, you will end up with the unit lag. (Figure 12).

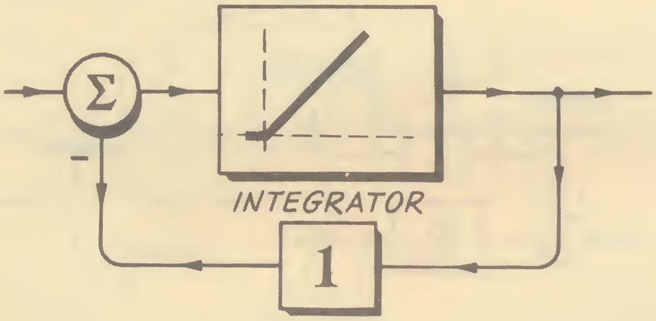


Figure 12. Unit Lag Element

So the ability of the use of feedback to transform one linear functional operator into another functional operator is once again made fairly evident by such comparisons.

A rather more dramatic pair occurs in the next figure, also linear operators, quite common to sociology, business, bioscience. Here is a time delay, fed back with a regeneration α of one, (i.e., a positive gain of unity), and it will produce what engineers are frequently wont to call a staircase integrator, and its mate shows its properties dramatically in Figure 14.

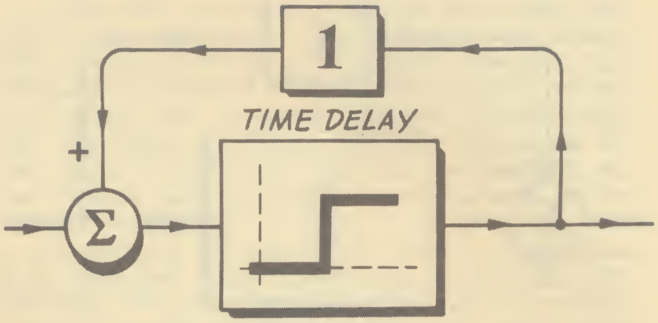


Figure 13. Staircase Integrator

In Figure 14 is the staircase integrator, fed back negatively now. It simply gives the time delay we started with. Perhaps it is a little hard to see, that this particular operator, fed back with a gain of minus one, would give us a time delay, but indeed it will, and again we do not need to perform the experiment to prove that this must be true.

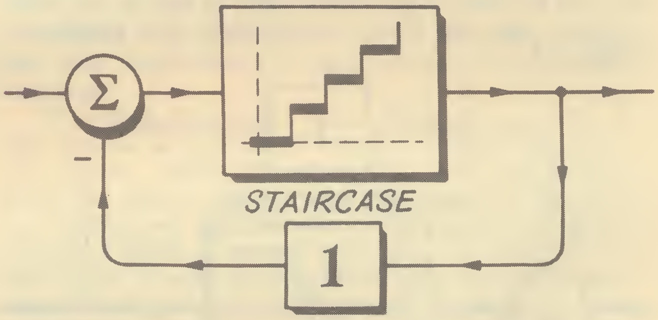


Figure 14. Time Delay Element

Combined Operators

Now I would like to show you the other thing that engineers in particular do with such concepts, by showing a typical hard and useful type of positive/negative feedback system. You will have then been given all the lessons necessary to appreciate the generality of the concepts under discussion.

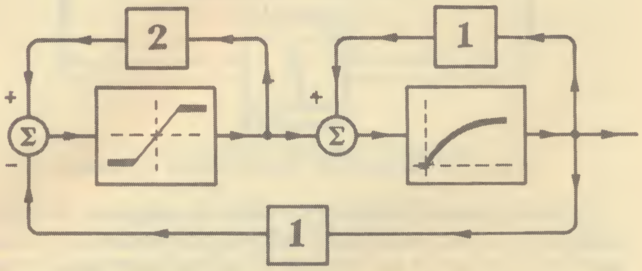


Figure 15. Typical (+/−) Feedback System

Again, our friend, saturating proportion, is fed back with a gain of plus two. If you are now trained to respond mechanically, you would tell me this is a hysteresis element, which indeed it is. This second functional element is a unit lag, an ubiquitous passive component, regeneratively fed back with gain plus one, to make an integrator. Therefore, the forward path yields an integrator, driven by a hysteresis element.

Now the final output is fed back unit negatively to the original summing point. What is the performance of this system? This system will generate at one point a triangular wave, and it will generate at another point a square wave.

Such generators can be made almost invisible in size, out of solid state components, and they are very useful as pacemakers, timekeepers, and other things. This is all done by taking two pieces of material which are themselves readily abundant, the first something that is proportional up to a saturating limit, and the second something which again exists ubiquitously in Nature: a passive lag.

This is an example of the deliberate use of both a global degenerative (negative) feedback, and also local regenerative (positive) feedback to accomplish a useful purpose, achieving a triangular and square wave oscillator.

I would like now to show a picture of an element with which I am abundantly familiar: a device to allow you use of both positive and negative feedback at any point in a system with any degree of dynamic or static interaction, (Figure 16).

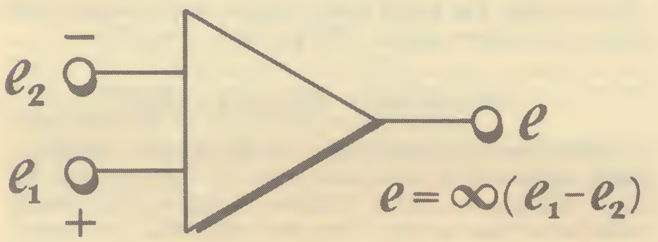


Figure 16. Differential Operational Amplifier

The output is indicated here symbolically as infinity of (or infinity times) the difference e_1 minus e_2, where e_1 is the *positive* input and e_2 is the *negative* input. The element is completely symmetrical in its behavior, except for the sign opposition of these two signals, but I hope you have absorbed the earlier message that there is a world of difference between minus and plus: about the same difference as between men and women!

Thus this element itself is a kernel element for a great many control and communication devices. Out of these as universal building blocks you can produce all of the systems that we mentioned, and many others.

Now the same or similar hardware shows up in commercial process controllers, and hydraulic servomechanisms, in electrical and mechanical elements of all sorts. The same signal philosophy is used throughout these areas. An observation of general philosophical and scientific significance is simply that great variety and flexibility of behavior can come out of extremely simple positive and negative feedback schemes.

Graphs and Abstractions

Before closing, it is of value to comment on the generality of the distinction between positive and negative feedback from the standpoint of linear graphs and abstract relations.

The topological structure of *any* system may be expressed as a graph consisting of curves (∿) and spots (• or ○), better called branches and nodes respectively. Significance usually attaches to a *directional* or *order* sense on the branches, denoted say by arrowheads (→ or ⟶). Finally, a unique mapping between directed and undirected branches exists if each undirected branch is replaced by a *duplex* pair of oppositely directed branches. That is [———] ≡ [⇄]. Shown nearby are representative undirected, directed and mixed graphs.

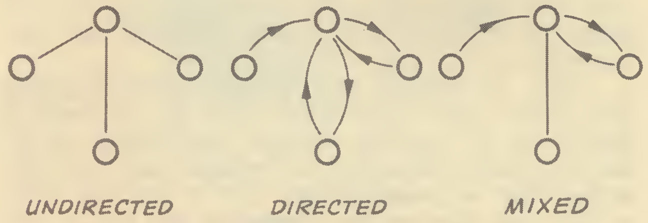


Figure 17. Typical Flow Graphs

Now a word as to the properties of the nodes of any graph. Most generally they might be unlabelled and therefore uniform and indistinguishable. However, in any directed graph, some of the branches incident on a given node will be directed toward it, others, away. *But it will always be observed that for unlimited richness of structure all graph systems must have at least one node-type with three or more branches and that if the graph is directed, at least two branches must be directed toward this node.*

Moreover, as the American philosopher and logician Charles Sanders Peirce discovered nearly a century ago, *but one triadic node-type, with two in-branches and one out-branch, is sufficient for infinite variety of structure.**

Clearly the quest for universal triads is at least as fruitful as that for the Philosopher's Stone or for the Holy Grail. At the level of binary logic, Peirce himself anticipated Sheffer in the discovery of the alternate denial and joint denial operators (the modern NOR and NAND). As may be readily established for this special case, a universal triad must be both associative/dissociative and symmetric with respect to in-branches.

Similarly, Mason's appreciation of summation as a universal triad led to his establishment of the signal flow graph as a mathematical system. In this case the nodes are the summing triads (or by association, polyads), the branches, causally-directed functional operators.

As a result of the above considerations, a directed linear graph, of sufficient extent, consisting solely of unlabelled nodes and directed branches, *wherein each node is a universal triad,* can represent, at the same time the most general possible Boolean logical system or, alternatively, the most general possible Mason signal flow graph. Brief reflection will show that all our previous results can be expressed in Mason graph form.

*Peirce's vivid sense of the importance of this concept is done justice only by his own words: "So prolific is the triad in forms that one may easily conceive that all variety and multiplicity of the universe springs from it . . . All that springs from the ⅄ — an emblem of fertility in comparison to which the holy phallus of religion's youth is a poor stick indeed." *(Collected Papers: 4.310)*

Finally, we may fruitfully enquire now as to the significance of feedback in such general cases. We observe that nearly all rich systems have corresponding rich feedback but that for the Boolean system (with NOR or NAND as the triad) no distinction relative to $(+/-)$ can be made. Conversely, for Masonian systems the $(+/-)$ distinction is possible precisely because simple summation has been taken as the universal triad.

Let me then make two final points on the role played by summation — by the summing point — in being able to discriminate between positive and negative feedback.

First, there could be a danger in misinterpreting the physical and engineering views too narrowly. I should say in physical sciences, particularly in the last ten years, the notion of feedback has been connected with commensurability and a common scale, in the sense that we are comparing two apples, two volts, or two feet. Indeed, a great deal of philosophical insight comes about as one searches for an additive, or superposible invariant, which will serve a broad class of systems. As a result of this quest, by restricting yourself to certain levels of additive invariance, you do arrive at great generalization of accomplishment. My own major interest has been for twenty years deliberate restriction to systems in which energy is the universal additive invariant; this covers a good deal of physics and the major part of engineering. Moreover, the superposition of energy is a prime requirement for it to have any meaning; if energy does not add, if it is not a scalar or additive invariant, it has no significance. This invariance goes right over into quantum and relativistic physics; indeed, it is the only aspect of energy which always continues valid, namely its simple superposition.

Now the second aspect of this question is that when you depart from mere addition, or simple superposition, you must look for another *structural triad* (in Peirce's terms): something in which two things combine to produce a third, or are related to a third. Here, the choice of the triad is very rich and broad. We have seen that in binary Boolean logics, for example, there are in fact *two* possible binary logical triads (NOR and NAND), either one of which, alone, is sufficient unto all binary logic. Thus all logical systems are but interconnected nets of such triads. Indeed, feedback still exists; there are still causal paths, yet now the significance of positive and negative disappears entirely; there is no meaningful way to distinguish which signal is positive and which is negative. The bisexual universe of male and female is thereby rendered hermaphroditic but no less prolix. But now strike out the triad and lo! — our glorious feedback is reduced to at best a lonely ouraboros.

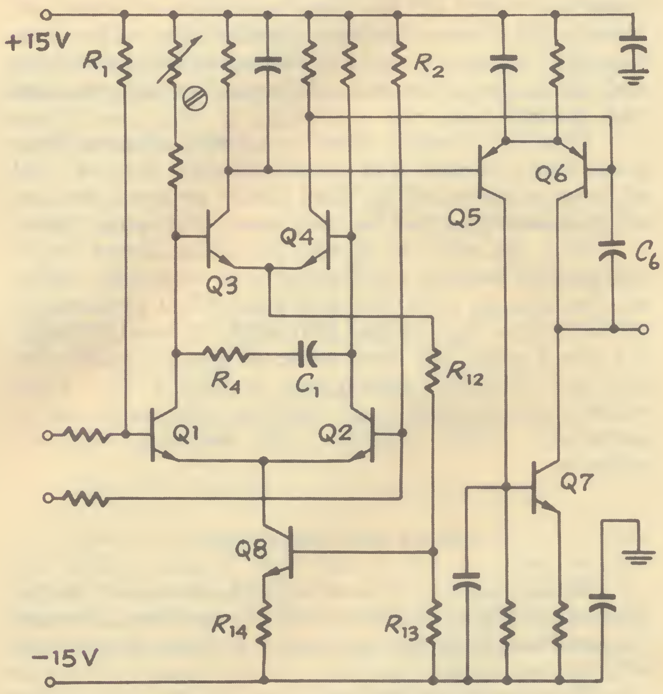
DESIGN OF A MODERN HIGH-PERFORMANCE OPERATIONAL AMPLIFIER

by Robert Allen Pease,
Vice President, and Manager, Engineering Dept.
Philbrick Researches, Inc.

The P85A arose from our need for a solid state operational amplifier with superior performance in differential applications, such as unity-gain followers and adder-subtractors. The special requirements for this kind of work are: high Common Mode Rejection Ratio, high common mode input impedance, and a capability for tolerating large, fast common mode signals without distortion. Of course, it should have all the essential features of a general-purpose operational amplifier like the P65A, such as high voltage and current gain, low drift (in terms of both voltage and current) and broad bandwidth, tempered by stability for resistive feedback of any degree. Also, in the interest of versatility, it should have the same basic plug-in configuration and pin connections as the P65A, P45A, and P35A.

Common Mode Operating Points

The essential character of the P85A's common-mode operation is provided by the configuration of the input transistors, which is shown in the accompanying schematic. The emitter current for Q1 and Q2 is provided by a constant-current-source comprised of Q8, and R12, R13, and R14. Consequently, the input transistors run at equal and constant collector currents independent of Common Mode voltage, within the range of ± 11 volts. The collector-emitter voltage of each transistor does, of course, vary with common-mode voltage, so that the "μ" of the input transistors



Schematic Diagram of Model P85A

must be well-matched to insure a good Common Mode Rejection Ratio. Fortunately, modern planar transistors (which have already been screened and matched for i_{cbo}, V_{ceo}, base-emitter voltage, and β) have very good, high, "μ" (which is $1/h_{rb}$) so that only a simple screening of the input pairs is necessary to obtain Common Mode Rejection Ratios of 40,000 to 300,000, as are typically observed.

Note further that no "feedforward" capacitors are connected to the input terminals of the P85A. Many solid state operational amplifiers employ capacitors connected from their inputs to their second or third stages, to insure stable broadband response and avoid spurious phase shifts attributable to input stages. The P85A does without these "feedforwards" in the interest of minimum input capacitance: 4 to 6 pF at either input, typically.

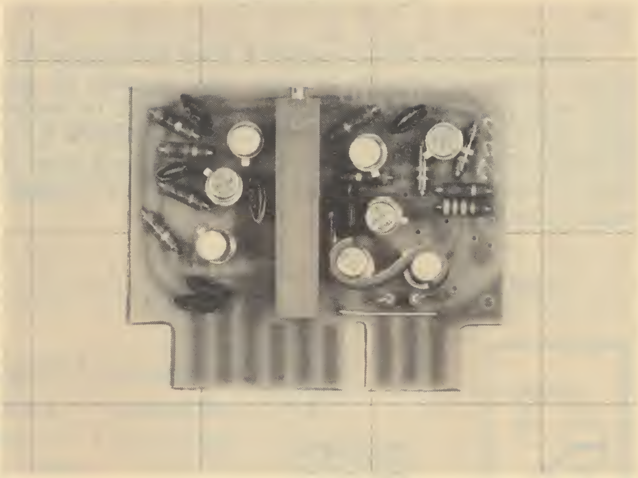
Another advantage derived from the absence of "feedforward" capacitors shows up whenever large, fast common-mode voltages occur. Even though the reactive currents drawn at the inputs by these "feedforward" capacitors may not be by themselves disastrous, the currents flowing at the other end of such capacitors may cause the stages to which they are connected to distort or saturate. This is a problem very hard to solve . . . except by avoiding the use of feedforwards! Consequently, the P85A can be used as an adder-subtractor even with common mode signals as large and fast as ± 10 volts at 100 kHz.

High common-mode input resistance is attained by the use of high-beta input transistors; high value resistors R1 and R2 are used to balance out the input transistors' base currents, and the impedance of the combination is typically 80 to 150 MΩ (60 MΩ minimum). The input stages of most amplifiers in the P85A family are protected by resistors of about 500 Ω value connected in series with each input base.

These resistors cause negligible errors (only a small number of microvolts) in ordinary operation, yet if a high voltage such as +15 V is accidentally connected to an input, no dangerous or destructive currents will flow, and the input transistors will be perfectly safe. Yet the amplifier can be used as a flipflop or multivibrator, with moderate differential voltages impressed between the inputs, without a "crowbar" effect, as would happen if side-by-side diodes were connected between inputs for protection.

Static Gain

Q1 and Q2 are a closely-matched pair, as has been mentioned, operating at moderately low collector currents, with a well-balanced load. The use of high-beta transistors for Q3 and Q4 insures a good voltage gain of about 15 for the input stage. The second stage operates again into a balanced load, and the use of moderate beta for Q5 and Q6 provides a decent voltage gain of about 20 for the second stage. The inherent symmetry of the P85A causes the voltage drift versus temperature to be very low — typically below 10 μV per degree centigrade. This low drift is attained with the help of a special (electrically insulated) thermal equalizer clip, which helps maintain the input transistors at equal temperatures. When one speaks of low voltage drifts of the order of 100 μV per 10°C or 100 μV per week, one should realize that the two input transistors' base-emitter voltages are typically 0.5 V, or 500,000 μV. Furthermore, the temperature coefficient of voltage



offset of either transistor is about 2 mV per °C, or 2,000 μV. Thus, we are talking about differential temperature stabilities of less than 1/20 °C. Of course, the dissipation in each input transistor must be, and is, kept very low, less than one-half milliwatt, to minimize heating effects.

Selection of transistors with high β, low $d\beta/dVce$, and low $d\beta/d\theta$, insure low DC input current, low current noise (typically 40 pA RMS for a 160 Hz to 16 kHz bandwidth), high common mode input impedance, and low current drift. A current drift from $-25°$ C to $+85°$ C of 200 nA is typically observed, while 440 nA is maximum value. (If especially high impedance levels are involved, the P85C, which has a special compensating network to minimize current drift versus temperature, should be specified.)

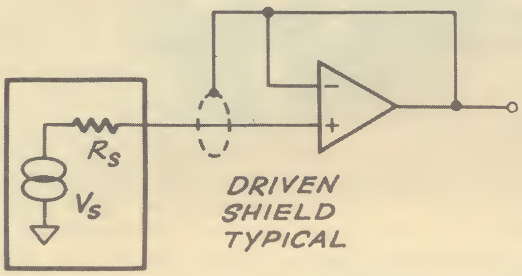
The output stage of the P85A uses a novel push-pull configuration, which draws on inherent balance to provide a voltage gain of 250 even at full rated load of ± 2.2 mA into a 5 kΩ load. This output configuration is inherently current-limiting, so that if the output is shorted to power ground, or to +15 V or −15 V supply, only about 3½ milliamperes will flow. Thus, neither the output stages, nor any delicate circuitry that might brush against the output, nor the power supply, will go up in smoke.

Dynamics

As we have already mentioned, no "feedforward" capacitors are used in the P85A. Consequently, each stage of gain must hold up predictably out beyond the unity-gain frequency of the amplifier, typically 3 MHz. The voltage gain of the first stage is rolled off by the network R4-C1 down to a value of about 1. This gain of 1 holds flat from about 3 kHz out beyond 10 MHz because transistors with very high current-gain-bandwidth are selected for Q3 and Q4. In this way, the apparent capacitance at the collectors of Q1 and Q2 is kept low, and does not have appreciable shunt effect on R4. The high-frequency voltage gain of the second and third stages is determined by the "Miller" feedback capacitor, C6. All the signal current Q4 delivers at its collector is integrated by Q6 and C6. The rolloff thus generated is inherently smooth at 6 db per octave. All the other capacitors are used simply as high-frequency bypasses.

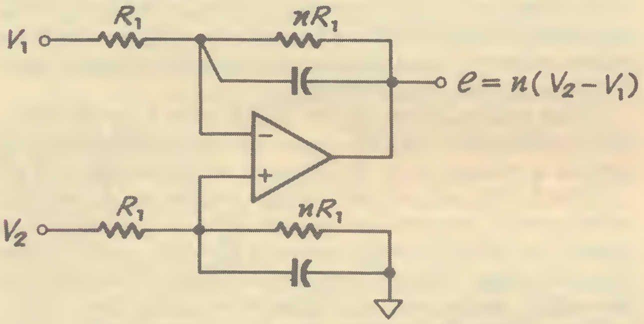
Some Applications

Naturally the P85A makes a good unity-gain follower. Input impedance is high, and because of the high voltage gain and CMRR, precision will be better than 0.01% of full scale for signals of impedance 2 kΩ or lower. Better than 1% accuracy is inherent even if source impedance rises as high as 500 kΩ, or if frequencies rise as high as 10 kHz.



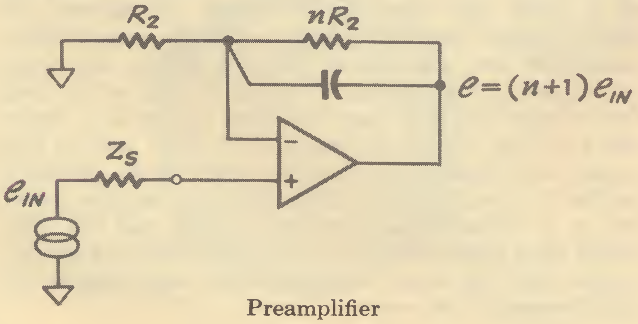
Unity Gain Follower

As an adder-subtractor, the P85A is capable of exquisite rejection of common-mode signals. If V1 and V2 are connected together, inherent rejection of low-frequency common mode signals is 80,000:1 typically, with 2 ppM nonlinearity. And high frequency common mode signals up to 100 kHz (even for ±10 V signals) can be rejected nicely when the indicated capacitors are trimmed.



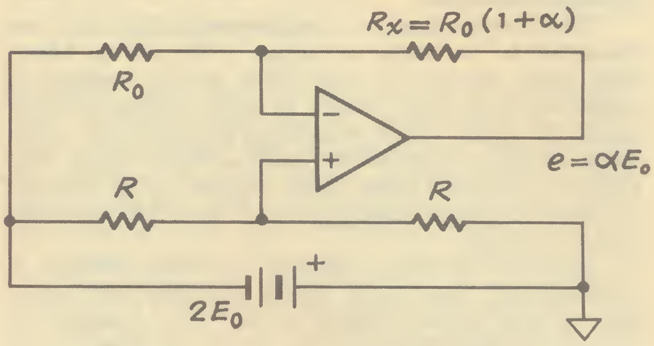
Adder-Subtractor

Because of the use of selected high-beta, low-noise transistors for the inputs, the P85A is ideal for use as a "high-fidelity" preamplifier, for DC and audio-frequency signals. A typical Noise Figure (for R_S = 20 k, Bandwidth = 20 Hz to 20 kHz), is 2 to 4 db. Voltage drift is typically 1 mV from −25° C to +85° C (3 mV maximum), and units certified for 1 mV maximum can be obtained also.



Preamplifier

This bridge circuit will give an output proportional to the deviation of the unknown from the standard, even for large deviations. The bridge supply is grounded and could be one side of the amplifier supply. The output is independent of the bridge impedance level, but to measure small deviations, another amplifier is required for good sensitivity.



Bridge Amplifier

An advantage afforded by the low input capacitance shows up when working with high-impedance signals at high frequencies. All other things being equal, an amplifier with lower input capacitance needs less feedback capacitor to insure dynamic stability. Thus, with less feedback capacitance, broader bandwidth can be attained, even at high impedance levels. In addition, lower input capacitance will make for a lower Noise Gain at moderate and high frequencies in high-impedance circuits, thus providing a better Signal-to-Noise ratio, even if and when increased bandwidth is enjoyed!

The P85A circuit is also available in the familiar cast-epoxy wire-in package as the PP85A. Current and voltage trims, if necessary, are provided by the user. The same basic design is also available in other versions, including low-cost utility, adjustable offset-current-compensated, low profile (0.4″ high), output-current boosted, and — with some modifications — in the transistor-sized low profile TO-8 "Q" package.

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